Worked example for the t-test

A group recorded the number of different kinds of herbaceous vascular plants in 2 areas of chalk grassland, which were originally managed identically. Now one is sheep-grazed every winter and one has not been grazed for 5 years.

20 quadrats each $0.1m^2$ were paced at random in each area and the number of kinds of plants (species richness) counted.

The group plotted rough frequency distribution histograms for the data and decided that the distribution was normal, allowing then to use the t-test

Null Hypothesis: there is no difference in species richness between the two areas
Alternative Hypothesis: there is a significant difference in species richness between the two areas

Observation number	Grazed area (x_1)		Ungrazed area (x_2)	
	x_1	x_1^2	x_2	x_2^2
1	14	196	6	36
2	18	324	5	25
3	12	144	6	36
4	16	256	8	64
5	12	144	7	49
6	13	169	5	25
7	13	169	6	36
8	16	256	8	64
9	14	196	9	81
10	9	81	9	81
11	10	100	10	100
12	11	121	3	9
13	13	169	5	25
14	15	225	8	64
15	12	144	7	49
16	15	225	6	36
17	14	196	9	81
18	13	169	8	64
19	14	196	6	36
20	11	121	5	25
Σ (sum)	265	3601	125	925
	Σx_1	Σx_1^2	Σx_2	Σx_2^2

Σ = the sum of, so to calculate the Σx_1, Σx_1^2, Σx_2 and Σx_2^2 values add up the values in each column.

Calculate the means of the x_1 and x_2 values to 3 decimal places:

$$\bar{x}_1 = \frac{\Sigma x_1}{n_1} = \frac{265}{20} \qquad \bar{x}_1 = 13.25$$

$$\bar{x}_2 = \frac{\Sigma x_2}{n_2} = \frac{125}{20} \qquad \bar{x}_2 = 6.25$$

Worked example for the t-test

So far we have calculated

Sum of x_1 values	Sum of squares of x_1 values	Sum of x_2 values	Sum of squares of x_2 values	Mean of x_1 values	Mean of x_2 values
Σx_1	Σx_1^2	Σx_2	Σx_2^2	$\bar{x}_1$	$\bar{x}_2$
265	3601	125	925	13.25	6.25

Now calculate the variance of each data set s_1^2 and s_2^2 to 3 decimal places in the boxes below.

$$s_1^2 = \frac{\Sigma x_1^2 - \frac{(\Sigma x_1)^2}{n_1}}{n_1 - 1} = \frac{3601 - \frac{265^2}{20}}{19} = 4.723$$

$$s_2^2 = \frac{\Sigma x_2^2 - \frac{(\Sigma x_2)^2}{n_2}}{n_2 - 1} = \frac{925 - \frac{125^2}{20}}{19} = 7.565$$

Calculate your t value by using the equation below (to 3 decimal places)

$$t = \frac{|\bar{x}_1 - \bar{x}_2|}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{7}{\sqrt{\frac{4.723}{20} + \frac{7.565}{20}}} = 8.930$$

Calculate your combined degrees of freedom $n_1 + n_2 - 2 = 18$

Now look up you critical value of t on the table below and compare it with your calculated t value

Critical value of t = 2.101
Calculated value of t = 8.930

So the calculated t value is **greater than** the critical value of t and we can reject your null hypothesis and accept the alternative hypothesis.

Critical values of t at the 5% probability level							
Combined degrees of freedom	**Critical value of t**	**Combined degrees of freedom**	**Critical value of t**	**Combined degrees of freedom**	**Critical value of t**	**Combined degrees of freedom**	**Critical value of t**
5	2.571	13	2.160	21	2.080	29	2.045
6	2.447	14	2.145	22	2.074	30	2.042
7	2.365	15	2.132	23	2.069	35	2.030
8	2.306	16	2.120	24	2.064	40	2.021
9	2.262	17	2.110	25	2.060	46	2.014
10	2.228	18	2.101	26	2.056		
11	2.201	19	2.093	27	2.052		
12	2.179	20	2.086	28	2.049		